

A GLOBAL LAGRANGIAN DESCRIPTOR APPLIED TO THE KUROSHIO CURRENT

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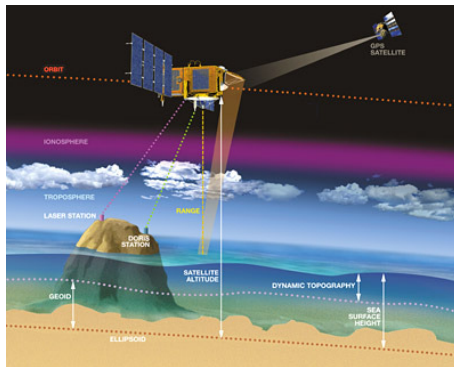
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September 26, 2011



ALTIMETRIC DATA

The altimeter at a satellite sends a wave to be analyzed after it is reflected on the sea surface. The level of the sea is thus measured as the difference of distances between the satellite–water surface (calculated from the lag the wave needs to return to the satellite) and the position of the satellite respect to a given reference ellipsoid close to the real surface. Besides the level of the sea, we can determine the height of the waves and the wind speed (from the amplitude y form of the reflected wave).

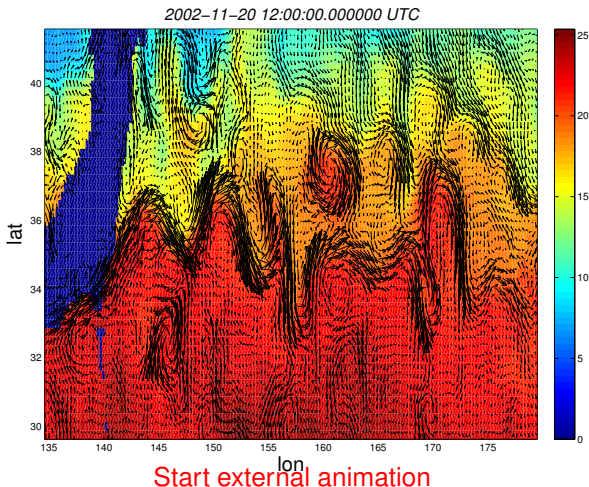


<http://www.aviso.oceanobs.com/es/altimetria/principio/index.html>



VELOCITY FIELD

Data have daily frequency and span from November 1 of 2002 to July 31 of 2003, with a resolution of $1/3^\circ$ in longitude (42 km in the Equator). Data come from project MERSEA 2008

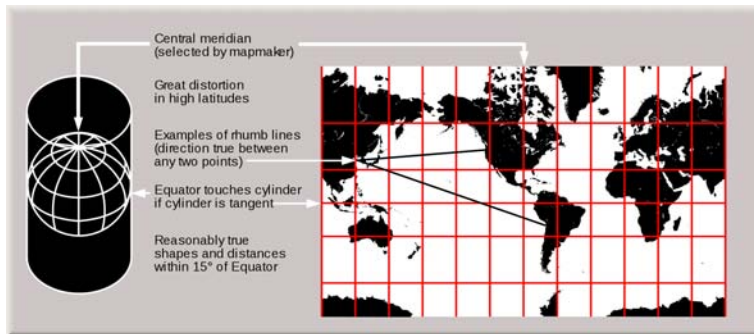


EQUATIONS OF MOTION

$$\begin{aligned}\frac{d\phi}{dt} &= \frac{u(\phi, \lambda, t)}{R \cos(\lambda)} \\ \frac{d\lambda}{dt} &= \frac{v(\phi, \lambda, t)}{R}\end{aligned}$$

ϕ means longitude and λ latitude.

MERCATOR PROJECTION



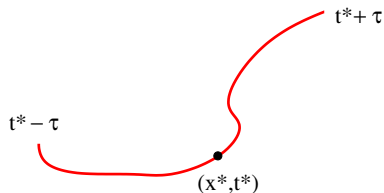
$$\mu = \ln |\sec \lambda + \tan \lambda|$$

FUNCTION M

Consider the dynamical system:

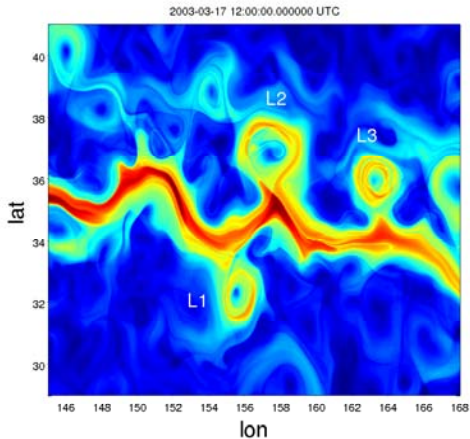
$$\frac{d\vec{x}}{dt} = \vec{v}(\vec{x}, t), \vec{x} \in \mathfrak{R}^2$$

where $\vec{v}(\vec{x}, t)$ is C^r ($r \geq 1$) in \vec{x} and continuous in t . Let $\vec{x}(t) = (x_1, x_2) \in \mathfrak{R}^2$ denote a trajectory. For all initial conditions \vec{x}^* in an open set $\mathcal{B} \in \mathfrak{R}^2$, at a given time t^* , we define the function $M(\vec{x}^*, t^*)_{\vec{v}, \tau} : (\mathcal{B}, t) \rightarrow \mathfrak{R}$ for the previous dynamical system as follows:



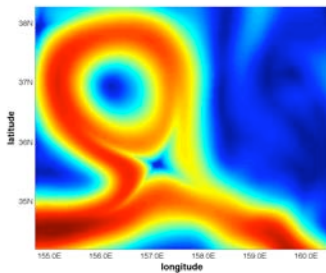
$$M(\vec{x}^*, t^*)_{\vec{v}, \tau} = \int_{t^* - \tau}^{t^* + \tau} \sqrt{\left(\frac{dx_1(s)}{ds}\right)^2 + \left(\frac{dx_2(s)}{ds}\right)^2} dt$$

FUNCTION M FOR MARCH 17 OF 2003

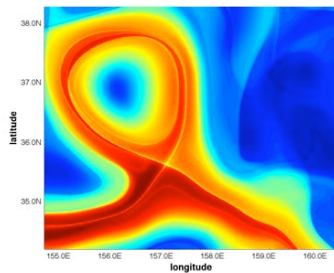


Start external animation

TIME τ



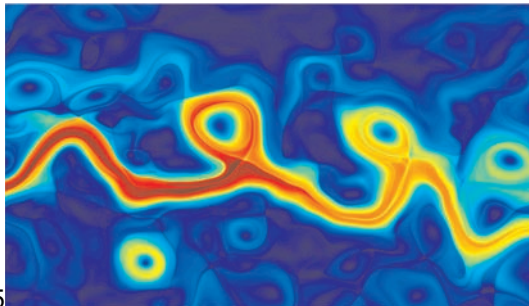
$\tau = 2$



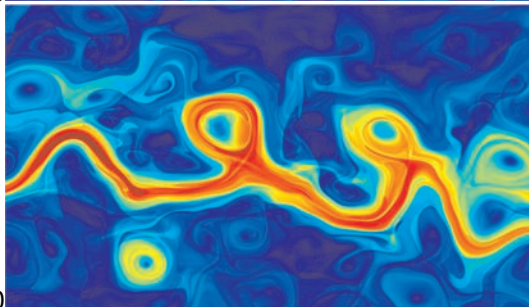
$\tau = 15$

FUNCTION M FOR MAY 2 OF 2003. LONGITUDE 148°E-168°E AND LATITUDE 30°N-41.5°N;

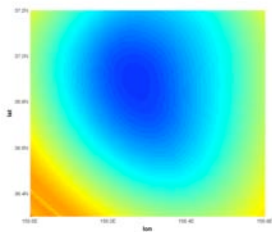
$\tau = 15$



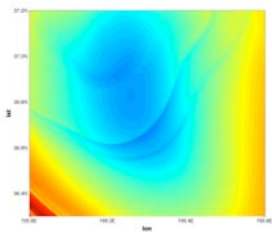
$\tau = 30$



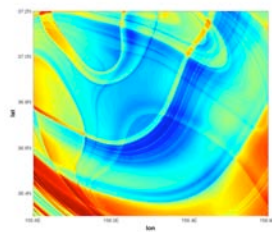
EVALUATION OF FUNCTION M AT THE CENTER OF AN EDDIE ON MAY 3 OF 2003



a) $\tau = 15$;

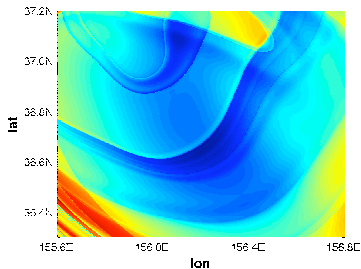


b) $\tau = 30$;

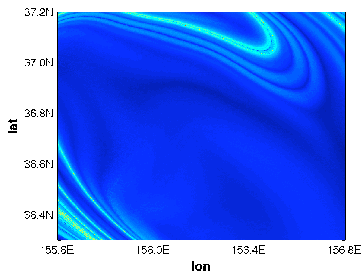
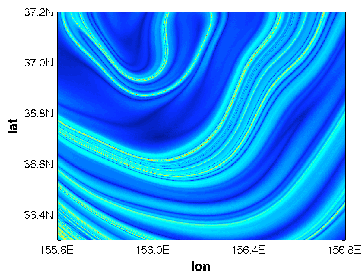


c) $\tau = 72$

FUNCTION M VS. LYAPUNOV EXPONENTS $\tau = 50$

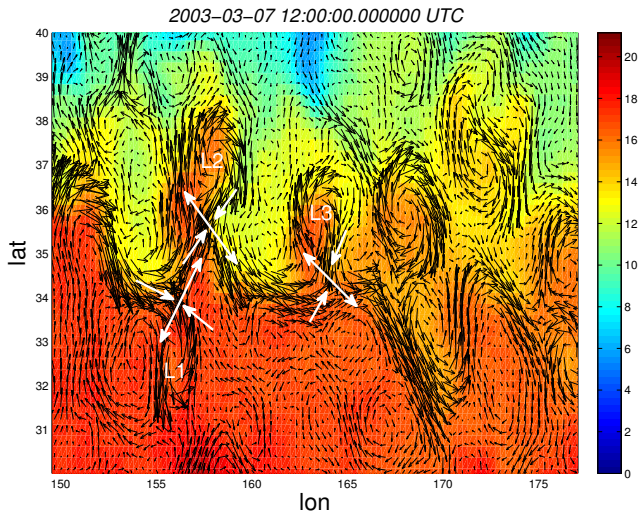


← Function M



← FTLE

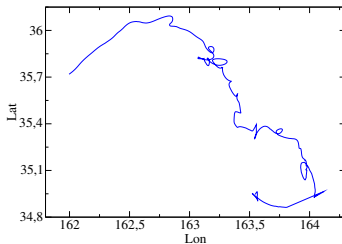
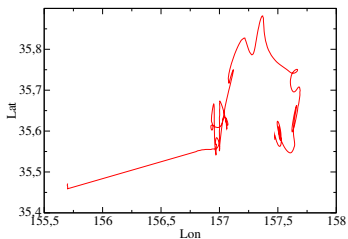
LIMIT COORDINATES



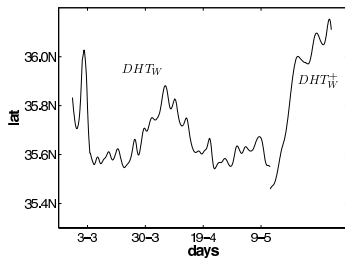
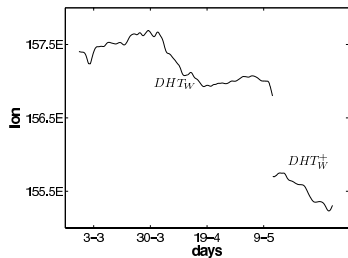
DISTINGUISHED TRAJECTORIES:

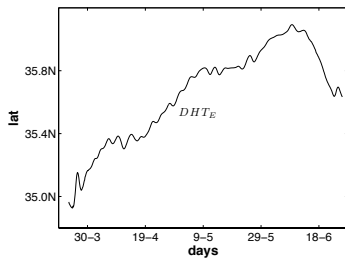
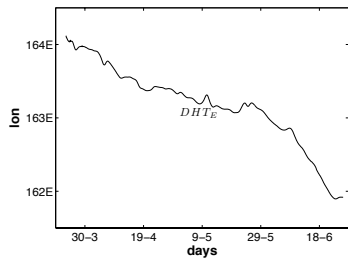
L2 $t = 113$ TO 184 AND L3 $t = 117$ TO 213

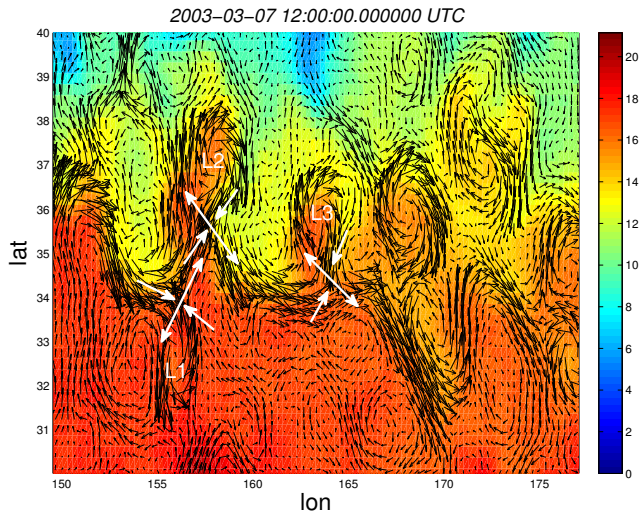
$$\frac{d\vec{x}}{dt} = f(\vec{x}, t) \rightarrow \vec{x} = \min(M(\vec{x}^*)_{t^*, \tau})$$



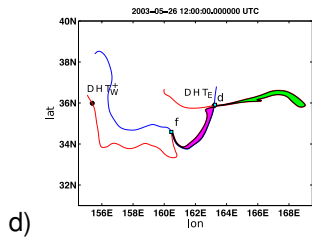
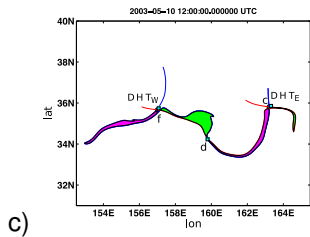
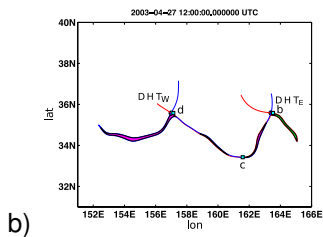
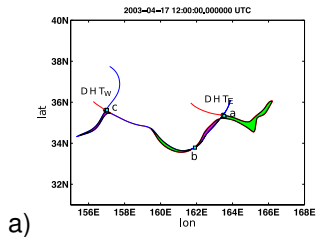
DISTINGUISHED HYPERBOLIC TRAJECTORIES (DHT)



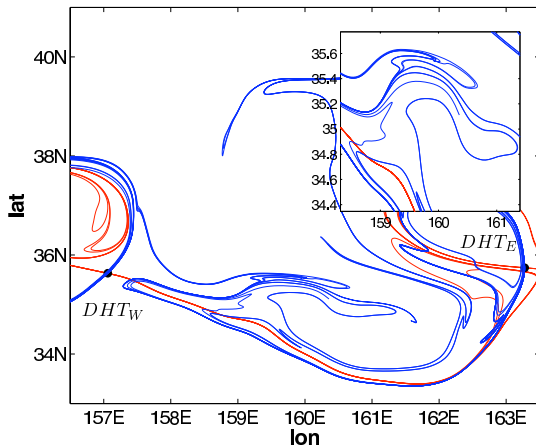




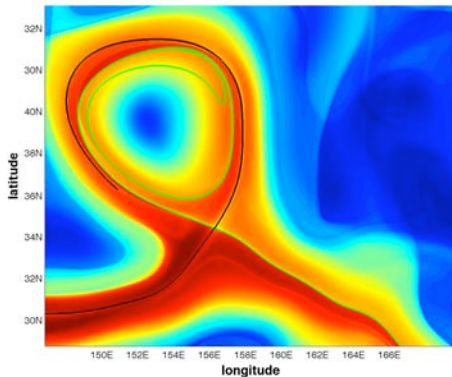
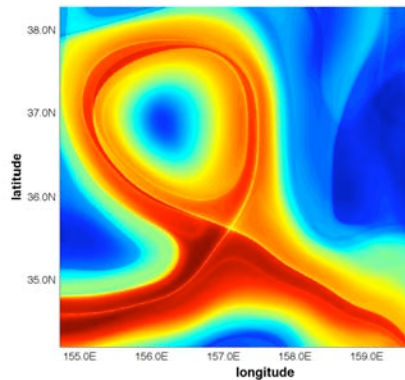
STABLE AND UNSTABLE MANIFOLDS OF TIME-DEPENDENT HYPERBOLIC TRAJECTORIES



2003-05-04 12:00:00.000000 UTC



$\tau = 15$ WITHOUT/WITH MANIFOLDS



- Unstable manifolds
- Stable manifolds
- Limit coordinates, white dots.

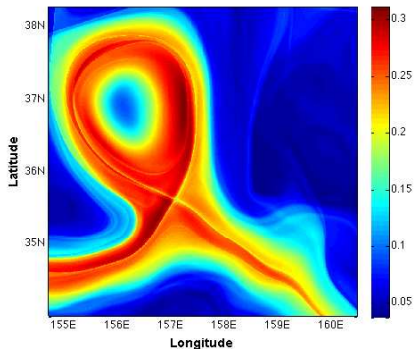
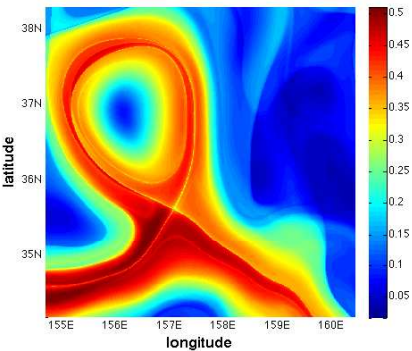
ACCELERATION

Integral of the modulus of the acceleration along trajectories:

$$|a(x(t))| = \sqrt{\sum_{i=1}^n \left(\frac{d^2 x_i(t)}{dt^2} \right)^2}$$

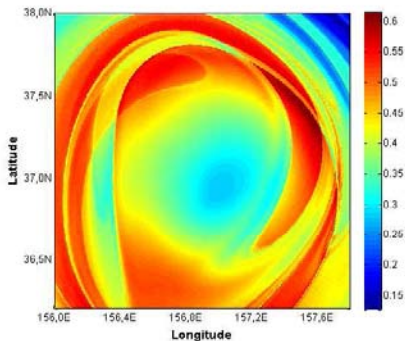
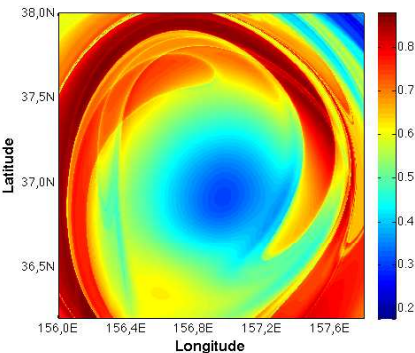
ARCLENGTH VS ACCELERATION

$$\tau = 15$$



ARCLENGTH VS ACCELERATION

$$\tau = 30$$



CONCLUSIONS

- We have calculated the function M and determined the Lagrangian skeleton of the Kuroshio current
- We have applied the definitions of limite coordinates to find the distinguished hyperbolic trajectories
- We have found that the evaluation of function M is a technique more precise and faster than the usual technique based on Lyapunov exponents